

# Quantum Creation of a Black Hole

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## Abstract

Using the Hartle-Hawking no-boundary proposal for the wave function of the universe, we can study the wave function and probability of a single black hole created at the birth of the universe. The black hole originates from a constrained gravitational instanton with conical singularities. The wave function and probability of a universe with a black hole are calculated at the *WKB* level. The probability of a black hole creation is the exponential of one quarter of the sum of areas of the black hole and cosmological horizons. One quarter of this sum is the total entropy of the universe. We show that these arguments apply to all kinds of black holes in the de Sitter space background.

PACS number(s): 98.80.Hw, 98.80.Bp, 04.60.Kz, 04.70.Dy

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## I. Introduction

Hawking's theory of No-Boundary Universe has for the first time led to a self-contained cosmology. Now in principle, one can predict everything in the universe solely from physical laws and the long-standing first cause problem has been dispelled. In quantum cosmology one of the most challenging problems is the existence of primordial black holes.

It is well known that a black hole can be formed in two ways, the first being the gravitational collapse of a massive star. If the mass of a star exceeds about twice that of the Sun, a black hole will be its ultimate corpse. The second way originates from the fluctuation of matter distribution in the early universe. In the big bang model, the matter content can be classically described [1][2], while in the inflationary universe the matter content is attributed to the quantum fluctuation of the Higgs scalar [3].

The discovery of Hawking radiation of black holes has been the most important event of gravitational physics for several decades. However, the life of the first kind of black hole is very much longer than the age of the universe. The only hope of confirming Hawking radiation by observation is through primordial black hole hunting.

Strictly speaking, black holes formed through either way mentioned above can hardly be regarded as primordial. A true primordial black hole should be created at the moment of the birth of the universe. Over the last decade there have been several attempts to deal with this problem, however their results are not conclusive [4][5].

It is believed that the very early universe is approximately described by a de Sitter metric. In quantum cosmology, at the Planckian era, the universe was created from a  $S^4$  space through a quantum transition. Therefore, to study the problem of primordial black hole creation in the de Sitter spacetime background is of twofold interest, for cosmology and for black hole physics.

There have been many studies recently on quantum creation of charged or neutral black hole pairs in the de Sitter spacetime background [6][7][8][9][10][11]. The case of a single primordial black hole is the topic of this paper. Sect. II is devoted to the theory of constrained gravitational instanton. Sect. III considers both the neutral and nonrotating black hole, i.e. the Schwarzschild black hole and the charged but nonrotating black hole, i.e. the Reissner-Nordström black hole. Sect. IV is devoted to the rotating but neutral black hole case, i.e. the Kerr black hole. Sect. V investigates

the rotating and charged case, i.e. the Newman black hole. By the no-hair theorem, all these kinds of black holes have exhausted the stationary vacuum or electrovac cases. Therefore, the problem of quantum creation of a single black hole in quantum cosmology is completely resolved. Sect. VI is a discussion.

## II. The constrained gravitational instantons

In the No-Boundary Universe the wave function of the universe is given by [12]

$$\Psi(h_{ij}, \phi) = \int_C d[g_{\mu\nu}] d[\phi] \exp(-\bar{I}([g_{\mu\nu}, \phi])), \quad (1)$$

where the path integral is over class  $C$  of compact Euclidean 4-metrics and matter field configurations, which agree with the given 3-metrics  $h_{ij}$  of the only boundary and matter configuration  $\phi$  on it. Here  $\bar{I}$  means the Euclidean action.

The Euclidean action for the gravitational part for a smooth spacetime manifold  $M$  with boundary  $\partial M$  is

$$\bar{I} = -\frac{1}{16\pi} \int_M d^4x g^{1/2} (R - 2\Lambda) - \frac{1}{8\pi} \int_{\partial M} d^3x h^{1/2} K, \quad (2)$$

where  $\Lambda$  is the cosmological constant,  $R$  is the scalar curvature,  $K$  is the trace of the second fundamental form of the boundary,  $g$  and  $h$  are the determinants of  $g_{\mu\nu}$  and  $h_{ij}$  respectively.

The dominant contribution to the path integral comes from some classical solutions of the field equations, which are the saddle points of the path integral.

The probability of the Lorentzian trajectory emanating from the 3-surface  $\Sigma$  with the matter field  $\phi$  on it can be written as

$$P = \Psi^* \Psi = \int_C d[g_{\mu\nu}] d[\phi] \exp(-\bar{I}([g_{\mu\nu}, \phi])), \quad (3)$$

where class  $C$  is all no-boundary compact Euclidean 4-metrics and matter field configurations which agree with the given 3-metric  $h_{ij}$  and matter field  $\phi$  on  $\Sigma$ .

Here, we do not restrict class  $C$  to contain regular metrics only, since the derivation from Eq. (1) to Eq. (3) has already led to some jump discontinuities in the extrinsic curvature at  $\Sigma$ .

The main contribution to the path integral in Eq. (3) is due to the stationary action 4-metric, which meets all requirements on the 3-surface  $\Sigma$  and other restrictions. At the *WKB* level, the

exponential of the negative of the stationary action is the probability of the corresponding Lorentzian trajectory.

From the above viewpoint, an extension of class  $C$  to include metrics with some mild singularities is essential. Indeed, in some sense, the set of all regular metrics is not complete, since for many cases, under the usual regularity conditions and the requirements at the equator  $\Sigma$ , there may not exist any stationary action metric, i.e. a gravitational instanton. It is not clear, how large class  $C$  should be. A necessary condition for a metric to be a member it that its scalar curvature should be well-defined mathematically. It is reasonable to include jump discontinuities of extrinsic curvature and their degenerate cases, that is the conical or pancake singularities. For this kind of singularity, the quantity  $g^{1/2}R$  can be interpreted as a distribution-valued density [13].

Although the regularity conditions on the 4-metrics and the requirements from the equator  $\Sigma$  sometimes are so strong that no gravitational instanton exists, one can still hopefully find a stationary action nonregular solution with some mild singularities within class  $C$ , which can be called the constrained gravitational instanton. Here the manifold of the instanton is constrained by the equator  $\Sigma$ .

It has been proven [13] a stationary action regular solution keeps its status under the extension of class  $C$ . However, if a stationary action regular solution cannot be found, then it can probably be expected with some singularities in class  $C$ . For a model with  $S^1 \times S^2$  topology under the minisuperspace ansatz

$$ds^2 = a^2(r)dr^2 + b^2(r)d\tau^2 + c^2(r)d\Omega_2^2, \quad (4)$$

where  $z$  is periodic with period  $2\pi$ , and  $d\Omega_2^2$  represents the metric of a unit 2-sphere, the solution satisfies the usual Einstein field equation except for the singularities at the final  $r = r_f$  and initial  $r = r_i$  surfaces. One can rephrase this by saying that the solution obeys the generalized Einstein equation in the whole manifold. Since this result is derived from first principles, one should not feel upset about this situation.

Except for the interpretation of probability the above arguments can also be applied to the Lorentzian regime with a purely imaginary phase. The dominating contribution is again due to the stationary action trajectories. However, in most cases, the restrictions are not too strong, and one can find a regular metric satisfying the usual Einstein field equation.

At the transition surface  $\Sigma$ , it is assumed that along neither of the sides of  $\Sigma$  does a singular matter distribution exist. It follows from the Einstein equation that the fundamental form  $K_{ij}$  at  $\Sigma$  should vanish,

$$K_{ij} = 0. \quad (5)$$

This condition cannot apply to the mild singularities at  $\Sigma$  if there is any, since the usual Einstein equation does not hold there.

The singularity problem associated with a gravitational instanton is not always disturbing; in fact it can be beneficial. If the restrictions are weak enough to allow a regular instanton, then the Lorentzian evolution originating from it must be most probable one. Therefore, in order to find the most probable Lorentzian evolution, one needs only find a regular instanton, which then identifies the 3-metric and matter field on  $\Sigma$ .

In general, the wave packet of a wave function of the universe represents an ensemble of classical trajectories. Under our scheme, the most probable trajectory associated with an instanton can be singled out [14]. Thus, quantum cosmology obtains its complete power of prediction. It means there is no more degree of freedom left as long as the model is well-defined.

On the other hand, the more severe the restrictions are, the larger the stationary action is, and therefore, the less probable its corresponding Lorentzian evolution. This is the situation with a constrained instanton. We shall see this in the case of a primordial black hole.

If there is no black hole in the universe, then one can get a regular instanton  $S^4$ . If there is, then the restrictions are strong enough to forbid regular solution. Therefore, the probability of a universe without a black hole is always greater than one with a black hole. Our calculation will support this.

There has been some progress in this direction. However, nearly all scenarios studied are associated with pair creation of black holes [6][7][8][9][10][11]. The main reason for this is that, people consider our universe to have been created by a quantum transition from a gravitational instanton. There does not exist any gravitational instanton which provides the seed for the creation of a single black hole in the de Sitter background.

As we mentioned above, in quantum cosmology one uses a Lorentzian metric to join a Euclidean metric, both being sectors of a complex manifold. However, there exist very few complex manifolds satisfying the Einstein equation with both a Euclidean and a Lorentzian sectors [15]. One may

appeal to some approximately Euclidean or Lorentzian sectors, but only at the price of losing some of the beauty of the theory. In the extended framework the requirement becomes quite loose. The situation of black hole creation we are going to investigate is the best illustration.

### III. The spherically symmetric black hole

Let us begin with a quantum spherically symmetric vacuum or electrovac model with a positive cosmological constant  $\Lambda$ . The cosmological

constant may be effective due to the Planckian inflation in the Hawking massive scalar model [16]. At the semiclassical level the evolution of the universe is described by its classical solutions. The Schwarzschild-de Sitter spacetime with mass parameter  $m$  and zero charge  $Q$  is the unique spherically symmetric vacuum solution to the Einstein equation with a cosmological constant  $\Lambda$ . The Reissner-Nordström-de Sitter spacetime, with mass parameter  $m$ , nonzero charge  $Q$  and a cosmological constant  $\Lambda$ , is the only spherically symmetric electrovac solution to the Einstein and Maxwell equations. Its Euclidean metric can be written as [17]

$$ds^2 = \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right) d\tau^2 + \left(1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (6)$$

We can set

$$V_s = 1 - \frac{2m}{r} + \frac{Q^2}{r^2} - \frac{\Lambda r^2}{3}. \quad (7)$$

For convenience one can make a factorization

$$V_s = -\frac{\Lambda}{3r^2}(r - r_0)(r - r_1)(r - r_2)(r - r_3), \quad (8)$$

where  $r_0, r_1, r_2, r_3$  are in ascending order.  $r_2$  and  $r_3$  are the black hole and cosmological horizons, where conical singularities may occur,  $r_0$  is negative. If the black hole is neutral, then  $r_1$  can be set to zero, and there are essentially three roots left.

The gauge field is

$$F = -\frac{iQ}{r^2} d\tau \wedge dr \quad (9)$$

for an electrically charged solution, and

$$F = Q \sin\theta d\theta \wedge d\phi \quad (10)$$

for a magnetically charged solution. We shall not consider dyonic solutions.

The roots satisfy the following relations

$$\sum_i r_i = 0, \quad (11)$$

$$\sum_{i>j} r_i r_j = -\frac{3}{\Lambda}, \quad (12)$$

$$\sum_{i>j>k} r_i r_j r_k = -\frac{6m}{\Lambda} \quad (13)$$

and

$$\prod_i r_i = -\frac{3Q^2}{\Lambda}. \quad (14)$$

The black hole and cosmological surface gravities  $\kappa_2$  and  $\kappa_3$  are [13]

$$\kappa_2 = \frac{1}{2}|V'_s(r_2)| = \frac{\Lambda}{6r_2^2}(r_2 - r_0)(r_2 - r_1)(r_3 - r_2), \quad (15)$$

$$\kappa_3 = \frac{1}{2}|V'_s(r_3)| = \frac{\Lambda}{6r_3^2}(r_3 - r_0)(r_3 - r_1)(r_3 - r_2). \quad (16)$$

The requirement of vanishing second fundamental form at  $\Sigma$  minus the two conical singularities at the two horizons implies that the transition can only occur at two sections of constant values of imaginary time  $\tau$  glued at the two horizons. The 3-surface  $\Sigma$  has topology  $S^2 \times S^1$ . To form a constrained gravitational instanton, one can have two cuts at  $\tau = \text{consts.}$  between  $r = r_2$  and  $r = r_3$ . Then the  $f_2$ -fold cover turns the  $(\tau - r)$  plane into a cone with a deficit angle  $2\pi(1 - f_2)$  at the black hole horizon. In a similar way one can have an  $f_3$ -fold cover at the cosmological horizon. Both  $f_2$  and  $f_3$  can take any pair of real numbers with the relation

$$f_2\beta_2 = f_3\beta_3, \quad (17)$$

where  $\beta_2 = 2\pi\kappa_2^{-1}$  and  $\beta_3 = 2\pi\kappa_3^{-1}$ . If  $f_2$  or  $f_3$  is different from 1, then the cone at the black hole or cosmological horizon will have an extra contribution to the action of the instanton. After the transition to Lorentzian spacetime, the conical singularities will only affect the real part of the phase of the wave function, i.e. the probability of the creation of the black hole.

Since the integral of  $K$  with respect to the 3-area in the boundary term of the action (2) is the area increase rate along its normal, then the extra contribution due to the conical singularities can be considered as the degenerate form shown below

$$\bar{I}_{2,deficit} = -\frac{1}{8\pi} \cdot 4\pi r_2^2 \cdot 2\pi(1 - f_2), \quad (18)$$

$$\bar{I}_{3,deficit} = -\frac{1}{8\pi} \cdot 4\pi r_3^2 \cdot 2\pi(1 - f_3). \quad (19)$$

The action due to the volume is

$$\bar{I}_v = -\frac{f_2\beta_2\Lambda}{6}(r_3^3 - r_2^3) \pm \frac{f_2\beta_2Q^2}{2}(r_2^{-1} - r_3^{-1}), \quad (20)$$

where  $+$  is for the magnetic case and  $-$  is for the electric case. This term disappears for the neutral case.

In the neutral case, the boundary data on the 3-surface  $\Sigma$  will be  $h_{ij}$ . In the magnetic case, the boundary data is  $h_{ij}$  and  $A_i$ . The vector potential in turn determines the magnetic charge, since it can be obtained by the magnetic flux, or the integral of the gauge field  $F$  over the  $S^2$  space sector. It is more convenient to choose a gauge potential

$$A = Q(1 - \cos\theta)d\phi \quad (21)$$

to evaluate the flux.

In the electric case, the boundary data is  $h_{ij}$  and the momentum  $\omega$  [11], which is canonically conjugate to the electric charge and defined by

$$\omega = \int A, \quad (22)$$

where the integral is around the  $S^1$  direction. The most convenient choice of the gauge potential for the calculation is

$$A = -\frac{iQ}{r^2}\tau dr. \quad (23)$$

The wave function for the equator is the exponential of half the negative of the action. For the neutral and magnetic cases, one obtains the wave function  $\Psi(h_{ij})$  and  $\Psi(Q, h_{ij})$ . For the electric



case, what one obtains this way is  $\Psi(\omega, h_{ij})$  instead of  $\Psi(Q, h_{ij})$ . One can get the wave function  $\Psi(Q, h_{ij})$  for a given electric charge through the Fourier transformation [10][11]

$$\Psi(Q, h_{ij}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega e^{i\omega Q} \Psi(\omega, h_{ij}). \quad (24)$$

This Fourier transformation is equivalent to a multiplication of an extra factor

$$\exp\left(\frac{-f_2\beta_2 Q^2(r_2^{-1} - r_3^{-1})}{2}\right) \quad (25)$$

to the wave function. This makes the probabilities for magnetic and electric cases equal, and thus recovers the duality between the magnetic and electric black holes [11].

Finally, using the relations (17) and (11)-(14), one obtains the probability for a spherically symmetric black hole creation

$$P_s \approx \exp(\pi(r_2^2 + r_3^2)). \quad (26)$$

This is the exponential of one quarter of the sum of the black hole and cosmological horizon areas, or the total entropy of the universe.

The most remarkable fact is that the result is independent of our choice of  $f_2$  or  $f_3$ . It means the manifold has a stationary action, therefore it can be qualified as a constrained gravitational instanton, and it can be used for the *WKB* approximation to the wave function. The same phenomenon will occur to the Kerr-Newman case as one will see later.

For the cases of the nonsingular, charged or neutral, spherically symmetric instantons and the associated black hole creations [6][7][8][9][10][11], all these instantons lead to the creation of pairs of black holes. For these cases one can avoid the conical singularities by choosing  $f_2 = f_3 = 1$ , since the two surface gravities are identical. However, their results are the special cases of our general formula (26), recalling that the degenerate horizon should be counted twice.

The wave function for the spherically symmetric black hole can also be found [5].

When  $m = 0$  and  $Q = 0$ , it is reduced to the de Sitter case

$$P_0 \approx \exp\left(\frac{3\pi}{\Lambda}\right) \quad (27)$$

and when  $Q = 0$  and  $r_2 = r_3$ , it is reduced to the Nariai case

$$P_{m_c} \approx \exp\left(\frac{2\pi}{\Lambda}\right). \quad (28)$$

The formula (26) interposes the above values for the two extreme cases of neutral black holes.

The probability is a decreasing function with respect to parameter  $m$  and  $|Q|$ . So the de Sitter universe is the most probable one for the Planckian era in quantum cosmology, as is expected.

#### IV. The Kerr-de Sitter black hole

Now let us discuss the creation of a rotating black hole in the de Sitter space background. The Lorentzian metric of the black hole spacetime is [17]

$$ds^2 = \rho^2(\Delta_r^{-1}dr^2 + \Delta_\theta^{-1}d\theta^2) + \rho^{-2}\Xi^{-2}\Delta_\theta \sin^2\theta(adt - (r^2 + a^2)d\phi)^2 - \rho^{-2}\Xi^{-2}\Delta_r(dt - a\sin^2\theta d\phi)^2, \quad (29)$$

where

$$\rho^2 = r^2 + a^2 \cos^2\theta, \quad (30)$$

$$\Delta_r = (r^2 + a^2)(1 - \Lambda r^2\mathfrak{Z}^{-1}) - 2mr + Q^2 + P^2, \quad (31)$$

$$\Delta_\theta = 1 + \Lambda a^2\mathfrak{Z}^{-1} \cos^2\theta, \quad (32)$$

$$\Xi = 1 + \Lambda a^2\mathfrak{Z}^{-1} \quad (33)$$

and  $m, a, Q$  and  $P$  are constants,  $m$  and  $ma$  representing mass and angular momentum.  $Q$  and  $P$  are electric and magnetic charges.

One can factorize  $\Delta_r$  as follows

$$\Delta_r = -\frac{\Lambda}{3}(r - r_0)(r - r_1)(r - r_2)(r - r_3), \quad (34)$$

where the roots  $r_0, r_1, r_2$  and  $r_3$  are in ascending order,  $r_2$  and  $r_3$  are the black hole and cosmological horizons. The roots satisfy the following relations:

$$\sum_i r_i = 0, \quad (35)$$

$$\sum_{i>j} r_i r_j = -\frac{3}{\Lambda} + a^2, \quad (36)$$

$$\sum_{i>j>k} r_i r_j r_k = -\frac{6m}{\Lambda}, \quad (37)$$

$$\prod_i r_i = -\frac{3(a^2 + Q^2 + P^2)}{\Lambda}. \quad (38)$$

In this section we shall concentrate on the neutral case with  $Q = P = 0$ . The Newman case with nonzero electric or magnetic charge will be deferred to the next section.

The probability of the Kerr black hole creation, at the *WKB* level, is the exponential of the negative half of its corresponding constrained gravitational instanton. The only instanton which can be used to join the Lorentzian sector at the quantum transition is the complex spacetime obtained from the Lorentzian metric by a substitution  $t \longrightarrow -i\tau$  only. However, for convenience of calculation, we can let  $a$  to be imaginary, and then the complex metric becomes Euclidean. After we get the probability for the imaginary  $a$  value, then we can analytically continue back to real  $a$  to obtain the required probability.

In order to form a constrained gravitational instanton, one can do the similar cutting, folding and covering at both the black hole and cosmological horizons with  $f_2$  and  $f_3$  satisfying relation (17) as in the nonrotating case. We shall freely switch back and forth between the real and imaginary values of  $a$  in the following calculation to facilitate our interpretation.

For the Kerr case, the topology of 3-surface  $\Sigma$  is  $S^2 \times S^1$ . Their horizon areas are

$$A_2 = 4\pi(r_2^2 + a^2)\Xi^{-1}, \quad (39)$$

$$A_3 = 4\pi(r_3^2 + a^2)\Xi^{-1}. \quad (40)$$

The black hole and cosmological surface gravities are

$$\kappa_2 = \frac{\Lambda(r_2 - r_0)(r_2 - r_1)(r_3 - r_2)}{6\Xi(r_2^2 + a^2)}, \quad (41)$$

$$\kappa_3 = \frac{\Lambda(r_3 - r_0)(r_3 - r_1)(r_3 - r_2)}{6\Xi(r_3^2 + a^2)}. \quad (42)$$

The actions due to the conical singularities are

$$\bar{I}_{2,deficit} = -\frac{\pi(r_2^2 + a^2)(1 - f_2)}{\Xi}, \quad (43)$$

$$\bar{I}_{3,deficit} = -\frac{\pi(r_3^2 + a^2)(1 - f_3)}{\Xi}. \quad (44)$$

The action due to the volume is

$$\bar{I}_v = -\frac{f_2\beta_2\Lambda}{6\Xi^2}(r_3^3 - r_2^3 + a^2(r_3 - r_2)), \quad (45)$$

where  $\beta_2$  is defined as before.

If one naively takes the exponential of the negative of half the total action (after the analytic continuation by the replacement of  $b$  by  $a$ ), then the wave function for the creation moment of a black hole with parameter  $m$  and  $a$  will not be obtained. The physical reason is that what one can observe is only the angular differentiation, or the relative rotation of the two horizons. This situation is similar to the case of a Kerr black hole in the asymptotically flat background. There one can only measure the rotation of the black hole horizon from the spatial infinity. To find the wave function for the given mass and angular momentum one has to make the Fourier transformation

$$\Psi(m, a, h_{ij}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\delta e^{i\delta J \Xi^{-2}} \Psi(m, \delta, h_{ij}), \quad (46)$$

where  $\delta$  is the relative rotation angle for the time period  $f_2\beta_2$ , which is canonically conjugate to the angular momentum  $J \equiv ma$ ; and the factor  $\Xi^{-2}$  is due to the time rescaling. The angle difference  $\delta$  can be evaluated

$$\delta = \int_0^{f_2\beta_2/2} d\tau (\Omega_2 - \Omega_3), \quad (47)$$

where the angular velocities at the two horizons are

$$\Omega_2 = \frac{a}{r_2^2 + a^2}, \quad (48)$$

and

$$\Omega_3 = \frac{a}{r_3^2 + a^2}. \quad (49)$$

The Fourier transformation is equivalent to adding an extra term into the action for the constrained instanton, and then the total action becomes

$$\bar{I} = -\pi(r_2^2 + a^2)\Xi^{-1} - \pi(r_3^2 + a^2)\Xi^{-1}. \quad (50)$$

It is crucial to note that the action is independent of  $\beta_2$ , and therefore we obtain the constrained instanton. The probability of the Kerr black hole creation is

$$P_k \approx \exp(\pi(r_2^2 + a^2)\Xi^{-1} + \pi(r_3^2 + a^2)\Xi^{-1}). \quad (51)$$

It is the exponential of one quarter of the two horizon areas, or the total entropy of the universe.

## V. The Newman-de Sitter black hole

Now let us turn to the charged black hole case. The vector potential can be written as

$$A = \frac{Qr(dt - a \sin^2 \theta d\phi) + P \cos \theta (adt - (r^2 + a^2)d\phi)}{\rho^2}. \quad (52)$$

We shall not consider the dyonic case below.

One can closely follow the neutral rotating case for calculating the action of the corresponding constrained gravitational instanton. The only difference is to add one more term due to the electromagnetic field to the action of volume. For the magnetic case, it is

$$\frac{f_2 \beta_2 P^2}{2\Xi^2} \left( \frac{r_2}{r_2^2 + a^2} - \frac{r_3}{r_3^2 + a^2} \right) \quad (53)$$

and for the electric case, it is

$$- \frac{f_2 \beta_2 Q^2}{2\Xi^2} \left( \frac{r_2}{r_2^2 + a^2} - \frac{r_3}{r_3^2 + a^2} \right) \quad (54)$$

In the magnetic case the vector potential determines the magnetic charge, which is the integral over the  $S^2$  space sector. Putting all these contributions together one can find

$$\bar{I} = -\pi(r_2^2 + a^2)\Xi^{-1} - \pi(r_3^2 + a^2)\Xi^{-1} \quad (55)$$

and the probability of the creation of a magnetically charged black hole is

$$P_n \approx \exp(\pi(r_2^2 + a^2)\Xi^{-1} + \pi(r_3^2 + a^2)\Xi^{-1}). \quad (56)$$

In the electric case, one can only fix the integral

$$\omega = \int A, \quad (57)$$

where the integral is around the  $S^1$  direction. So, what one obtains in this way is  $\Psi(\omega, a, h_{ij})$ . In order to get the wave function  $\Psi(Q, a, h_{ij})$  for a given electric charge, we have to repeat the

procedure like the Reissner-Nordström case. The Fourier transformation is equivalent to adding one more term to the action

$$\frac{f_2 \beta_2 Q^2}{\Xi^2} \left( \frac{r_2}{r_2^2 + a^2} - \frac{r_3}{r_3^2 + a^2} \right). \quad (58)$$

Then we obtain the same formula for the electrically charged rotating black hole creation as that for the magnetic one,

$$P_n \approx \exp(\pi(r_2^2 + a^2)\Xi^{-1} + \pi(r_3^2 + a^2)\Xi^{-1}). \quad (59)$$

It is easy to show that the probability is an exponentially decreasing function of the mass parameter, charge magnitude and angular momentum, and the de Sitter spacetime is the most probable Lorentzian evolution at the Planckian era.

## VI. Discussion

The result of this paper has shown that the probability of the black hole creation is the exponential of the total entropy of the universe. The entropy is equal to one quarter of the sum of the black hole and cosmological horizon areas.

The probability is an exponentially decreasing function in terms of the mass parameter, charge magnitude and angular momentum. Since this is only the confirmation of the conjecture, the result is no surprise. The only surprise is the fact that our result is independent of the choice of  $f_2$  or  $f_3$  for the formation of the constrained gravitational instantons.

To get a meaningful result, one has to be careful to identify the meaning of the wave function; so for the rotating case and electrically charged black holes, one has to introduce Fourier transformations into the calculation; otherwise the result becomes meaningless. It is interesting to note that Nature would give us a beautiful result if our request is reasonable.

In quantum field theory, the temperature associated with a black hole is well defined. By using the reciprocal of the Hawking temperature as the period of the imaginary time, one can avoid the conical singularity at the horizon. However, if we remain only at thermodynamics level, and if one considers the reciprocal of the period for the constrained gravitational instanton as an effective temperature, then from the calculation, it seems the temperature can be taken quite arbitrarily. We

appear to overcome the obstacle that the temperature of the black hole and cosmological horizons, in general, are different. This makes our calculation feasible. Temperature is a very subtle concept even in special relativity, let alone in general relativity. A thorough discussion about temperature is beyond the scope of this paper. However, the concept of entropy is very clear in any case.

Our calculation has also very clearly shown that the gravitational entropy is associated with topology of spacetime, as Hawking emphasized many times [18].

From the no-hair theorem, a stationary black hole in the de Sitter spacetime background can only have three parameters, mass, charge and angular momentum, so the problem of the quantum creation of a single black hole at the birth of the universe is completely resolved.

#### **Acknowledgment:**

I would like to thank Prof. G. Coyne of Specola Vaticana and Prof. R. Ruffini of Rome University for their hospitality.

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